# Accurate Representation of Arbitrary Depth Source Terms in Coastal Wave Prediction Models

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Award #s: N0001401MP20013 and N0001401MP20014

# **LONG-TERM GOALS**

The principal goal of this project is to improve our understanding of the interactions governing the spatial and temporal evolution of surface waves in arbitrary depths. This will be accomplished through the Shoaling Waves large-scale field experiment (SHOWEX) and coordinated wave modeling improvements through the Advanced Wave Prediction Program (AWPP). The results of these studies will fill the Naval Operational wave forecasting needs for accurate and computationally efficient estimates of the nonlinear wave-wave interaction (S<sub>nl</sub>) source term for arbitrary depths.

# **SCIENTIFIC OBJECTIVES**

The principal objective of this project is to investigate via numerical means the source term balance in shoaling waves, test newly constructed exact  $S_{nl}$  solutions, develop an improved Discrete Interaction Approximation (DIA, Hasselmann et al 1985; Komen et al 1994), test the approximation method, and ultimately implement this approximation in existing Naval Operational Wave Forecasting methodologies.

# **APPROACH**

The action balance equation has two distinct parts to be solved: spatial changes in the spectrum (i.e. propagation shoaling and refraction) and, the temporal changes described by the source terms: atmospheric input ( $S_{in}$ ), nonlinear wave-wave interaction ( $S_{nl}$ ), dissipation due to white-capping ( $S_{ds}$ ), and wave-bottom effects ( $S_{w-b}$ ). The Shoaling Waves field experiment (SHOWEX) will attempt to directly measure the atmospheric input, white-capping, and the effects of wave-bottom interactions. The nonlinear interactions will be directly calculated in this study using the work of Resio et al (2001) for arbitrary depths. Assessment of the dissipation measurements will be indirectly validated from the source term balance, (again computed form all mechanisms). Exact solutions (using the full dispersion relationship) as well as approximations derived from Herterich and Hasselmann (1980) scaled to deep water  $S_{nl}$  (Resio et al 2001) will be implemented in  $3^{rd}$  generation wave modeling technologies. Laboratory experiments conducted at the USAE Engineer Research and Development's Coastal and Hydraulics Laboratory (CHL) directional wave basin, will augment the SHOWEX data, isolating the role of  $S_{nl}$  and  $S_{ds}$  for time evolution of the directional wave spectra excluding the presence of the atmospheric source.

#### WORK COMPLETED

Pursuing a replacement for the DIA in existing Navy operational wave modeling technologies has focused on reducing the computational load required for the exact Webb-Resio-Tracy (Resio et al 2001) or the WRT method. Three methods have been pursued. The first method conditionally relaxes the computational space decreasing the frequency/directional resolutions. The second method takes advantage of new High Performance Computing architectures and tools for the implementation on scalable platforms. The third method focuses on the synthesis of a large number of exact WRT simulations for various input spectra to determine if a statistically coherent form of  $S_{nl}$  can be described in frequency and directional space using a large population of measured directional spectra.

#### **RESULTS**

The most computational intensive portion of all exact Boltzmann integral solution methods for nonlinear four-wave interactions is in the calculation of the coupling coefficients. Reducing the domain for these calculations, and/or reducing the resolution in the integral space would effectively reduce the computational burden. This reduction will impact the  $S_{nl}$  accuracy. Through the analysis we have found the WRT method is extremely sensitive to the frequency/directional resolution for the spectral shape and wind scenarios. Interactions in the neighborhood of the spectral peak are strong. A high-resolution frequency domain is required adequately represent the interactions around the spectra peak and to preserve the "+/-/+" signature of the  $S_{nl}$ . A slight reduction in the frequency resolution (despite weighting heavily around the spectral peak) will promote multiple interactions, and eventually cause temporal growth rates to diverge from well-established growth rate expressions. Use of new computational tools on high performance architectures and algorithms in the integration of  $S_{nl}$  reduced run times by two orders of magnitude. This is a great improvement, but scaling this to operational needs would necessitate additional two orders of magnitude improvement. This work was abandoned because it would compromise the overall accuracy of the WRT  $S_{nl}$  estimates.

The third focused effort involves the synthesis of many runs using the WRT method for a large population of measured directional wave spectra. Using multivariate data analysis techniques (canonical correlation) a functional relationship between the input spectra and the resulting  $S_{nl}$  can be generated. The canonical correlation is the maximum correlation between linear functions of the two vector variables (namely the input spectrum and the nonlinear wave-wave interaction solution for that spectrum). One can think of this as a multidimensional mapping that maximizes the correlation between the input function and its results. The relationship between the input spectra and S<sub>nl</sub> are apparent. There is a strong positive transfer of energy around the spectral peak, a negative lobe in the rear face, followed by a positive lobe at the higher frequencies. If the canonical correlation can successfully approximate the maximum positive, negative lobes, identify the location of those lobes in the frequency domain and possess similar integral quantities of the principal positive and negative lobes a replacement for the DIA could be realized. The range in wave conditions varies from 1.0-m (the cutoff for the analysis) to slightly over 5.0-m. Peak spectral wave periods range from 5-sec, to nearly 15-sec. Encompassing the nearly 500 individual cases are, wind-wave growth scenarios, mixed wind-seas and swell, and persistent swell. Results for the peak positive and negative lobes are presented in Figure 1, accompanying the results (red symbols), a linear regression and bin averaging (green line and green symbols) with one standard deviation identified is presented. The results indicate that the canonical correlation estimate for the peak positive magnitude represents the exact S<sub>nl</sub> WRT method through three decades (from 10<sup>-6</sup> to 10<sup>-3</sup>). The canonical correlation results depart from the line of perfect fit for values less than 10<sup>-6</sup>. These conditions are generally correlated to very low wave

steepness conditions, where the S<sub>nl</sub> would be less important. The maximum negative lobe results derived from the canonical correlation estimator are generally over estimated compared to the exact solution for the range from  $-10^{-3}$  to  $-10^{-6}$ . The consistent bias can be rectified through a modest adjustment in the coefficients via the results of the bin averaging. The results for the strength in the positive and negative S<sub>nl</sub> quantities parallel the findings in Figure 1. There is a greater tendency to underestimate the strength of the positive  $S_{nl}$  lobe than in the estimation of the maximum. This is caused by a persistence of the canonical correlation results to place a negative low frequency lobe in the S<sub>nl</sub> for non-uniformly increasing forward face spectra. The canonical correlation results depicting the strength of the negative lobe are slightly elevated as in the case of the maximum, and show less frequency randomness as in its positive counterpart. Estimating the location of the positive, negative and crossover (where  $S_{nl} = 0$ .) locations in the frequency domain is equally important as in the approximation of the strength. Of the three locations, the canonical correlation results are consistently better at the estimation of the positive peak followed by the negative peak (Figure 2). The crossover location, despite the favorable agreement between individual conditions displays more randomness in the bin averaging. By selectively filtering the time-periods where the canonical correlation misrepresents the peak positive magnitude, the frequency results become more consistent with the exact S<sub>nl</sub> estimates. For pure wind-seas, with relatively strong wave steepness, the canonical correlation estimates for the location of the peak positive, negative, and crossover frequencies are well represented.

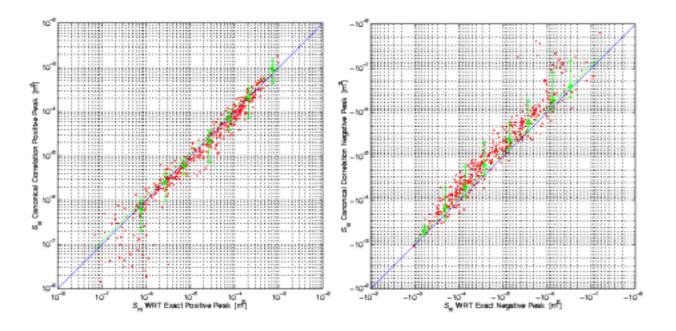


Figure 1. Canonical correlation versus exact WRT  $S_{nl}$  estimates for peak positive (left), and peak negative (right) magnitudes.

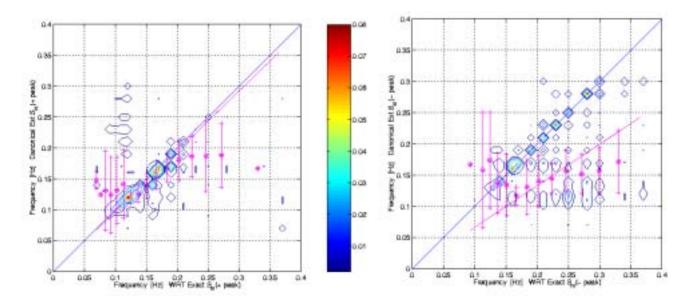


Figure 2. Canonical correlation versus exact WRT  $S_{nl}$  estimates for peak positive (left), and peak negative (right) locations in frequency domain.

# **IMPACT**

One views the continental shelf as an environment that significantly alters the deep-water directional wave spectrum. The source/sink terms impact their control over changes in the directional spectrum while bathymetric effects attempt to steer the energy dictated by local water depth gradients. What has been found thus far is that the arbitrary depth WRT  $S_{nl}$  is extremely sensitive to the frequency banding that will preclude inclusion of a less accurate, yet more computational efficient routine. The canonical correlation techniques shows promise in reproducing the exact solution over a wide range of real input conditions, while its computational demands are less restrictive than that of the existing DIA. The net gains in the correct estimation of  $S_{nl}$  as well as the basis to make significant strides in replacing the DIA with far more accurate approximation are now within reach.

# **TRANSITIONS**

The results derived from this study will be further developed in a transect model containing the complete set of source terms as well as the effects due to changing water depths. This model will be used to calculate source term balances for the SHOWEX project and used for selective testing of the CHL laboratory investigations. The WRT method will be incorporated in the architecture for research purposes and evaluated for deep and arbitrary depths. A total of over 1000 individual tests have been initiated with various frequency/direction separation limits, linear and abrupt directional shear cases, combinations of various  $\alpha$  and  $\gamma$  (Phillips equilibrium constant, and spectra peakedness parameters). The results of the canonical correlation estimation will be further pursued in the directional domain, developed and tested in the confines of 3<sup>rd</sup> generation wave modeling architectures. Ultimately, the results from these projects will yield a newly formulated approximation to  $S_{nl}$  to be ingested in Naval Operational Wave Forecasting Systems for better approximations of wave conditions over the continental shelf.

# RELATED PROJECTS

Listed below are various projects that are directly related to the SHOWEX and AWPP.

1. Headquarters, U.S. Army Corps of Engineers: "Improving Wave Estimates for Coastal Waters." Laboratory experiments in the directional wave flume, for the investigation, validation of modeling technologies and transition to the U.S. Army Corps district, division offices and in-house CHL staff for use in the estimation of wave conditions in the nearshore domain.

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